N = 1 Field Theory Duality from M-theory

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Abstract

We investigate Seiberg's $N = 1$ field theory duality for four-dimensional super-symmetric QCD with the M-theory 5-brane. We find that the M-theory configuration for the magnetic dual theory arises via a smooth deformation of the M-theory configuration for the electric theory. The creation of Dirichlet 4-branes as Neveu-Schwarz 5-branes are passed through each other in Type IIA string theory is given a nice derivation from M-theory.

1 Introduction

In the last few years there has been tremendous progress in understanding supersymmetric gauge dynamics and the remarkable phenomenon of electric-magnetic duality [1, 2]. Most of the results were first guessed at within field theory and then checked to satisfy many non-trivial consistency conditions. However, the organizing principle behind these dualities has always been somewhat mysterious from the field theoretic perspective.

Recently there has arisen a fascinating connection between supersymmetric gauge dynamics and string theory brane dynamics [3] which has the potential for unifying our understanding of these dualities [4-13]. This stems from our ability to set up configurations of branes in string theory with supersymmetric gauge field theories living on the world-volumes of branes in the low-energy limit. The moduli spaces of the gauge field theories are thereby encoded geometrically in the brane set-up. Furthermore, in several cases it has been shown that the magnetic dual of a field theory can be obtained as the low-energy limit of a brane configuration obtained by a continuous deformation of the brane configuration corresponding to the electric theory. This constitutes a derivation of field theoretic duality from string theory if the infrared limit is unaffected by the deformation. An alternative approach that allows one to derive non-trivial field theory results including $N = 1$ dualities has been developed using F-theory [14-21].

The strong coupling dynamics of the low-energy limit of supersymmetric gauge theories can also be studied using M-theory. In this approach the Dirichlet 4-branes (D4-branes) appearing in Type IIA string theory constructions of field theories, are replaced by M-theory 5-branes wrapped around the compact eleventh dimension, while the Neveu-Schwarz (NS) 5-brane of Type IIA string theory remains a 5-brane of M-theory. It has been shown that four-dimensional $N = 2$ supersymmetric gauge dynamics can then be represented in M-theory by a single 5-brane surface, and this surface is directly related to the curves that appear in the solutions to the Coulomb branch of the field theory [22] [23]. Subsequent developments for $N = 2$ appear in refs. [24 – 26]. The M-theory approach has also been generalized to the study of moduli space in $N = 1$ supersymmetric gauge theories [27,28]. In the context of pure $N = 1$ supersymmetric Yang-Mills theory, Witten has shown how the low-energy effective superpotential can be computed from M-theory [29]. This was subsequently generalized to the case including matter in ref. [30]. Witten also derived a hadronic string from a special limit of M-theory, and this has also been generalized to the case with matter [31].

In this paper we derive Seiberg's dualities for four-dimensional $N = 1$ supersymmetric QCD (SQCD) from M-theory. A central result is that both the SQCD electric theory and its magnetic dual (when there is one) are described by the same M-theory configuration. Specifically, one can start from the string theory brane configuration whose low-energy limit is the electric theory, make the string theory coupling large

enough to pass into M-theory, change the parameter in the M-theory configuration which corresponds to the (electric) strong interaction scale from very small to very large (compared to the string scale), and then make the string coupling weak again. The result is the string theory brane configuration whose low-energy limit is the magnetic dual field theory! The advantage of our M-theory derivation of duality over the corresponding string theory derivations is that the presence of the compact eleventh dimension of M-theory smoothes the singular situations that can occur as intermediate steps in the deformation of the electric configuration to the magnetic configuration in string theory. By going to M-theory we derive a pleasing picture of how D4-branes are created when NS 5-branes are passed through each other.

The organization of this paper is as follows. Section 2 reviews those features of SQCD which we use in the rest of the paper, and sets some notation. Section 3 reviews the string theory brane configuration whose low-energy limit is SQCD. Section 4 describes the translation of this set-up to M-theory, largely following ref. [28]. Section 5 shows how SQCD duality emerges from the M-theory description, with a minor technical flaw associated with our use of semi-infinite D4-branes. This flaw is corrected in section 6, by using only finite D4-branes, in the special case of equal quark masses. An interesting associated subtlety which is resolved by non-perturbative bending of D4-branes is discussed. Section 7 provides our conclusions.

2   Review of SQCD

Detailed derivations of the results quoted here can be found in refs. [1, 2]. Consider an $N = 1$ SQCD theory with $SU(N)$ gauge group and F flavors of quarks (Q_+) and anti-quarks (Q_-). The standard (one-loop) formula for the strong interaction scale is given by

$$\Lambda^{3N-F} = \mu^{3N-F} e^{-(8\pi^2/g_{SQCD}^2(\mu)+i\theta)}, \tag{2.1}$$

where the phase is given by the CP-violating θ angle.

The quark mass matrix is denoted by m, and (for technical string/M-theory reasons) we will restrict our attention to the case where the mass eigenvalues, $m_1, ..., m_F$ are all non-vanishing. They can however be chosen to be arbitrarily small and can then be usefully thought of as sources for quark bilinears in massless SCQD. If however, we take a quark mass to be very large, say m_F, we can integrate out the massive quark and get a lower-energy effective theory with $F - 1$ flavors and a strong scale given by

$$\Lambda_L^{3N-F+1} = m_F \Lambda^{3N-F}. \tag{2.2}$$

The masses lift all the classical flat directions of massless SQCD, which are parameterized by meson, (and for $F \geq N$) baryon and anti-baryon chiral superfields. As a consequence, the baryon and anti-baryon vevs are fixed at zero, while the vev of the

meson fields defined by the gauge-invariant bilinear,

$$M \equiv Q_+ Q_-, \qquad (2.3)$$

is diagonalized in the same basis as m, with eigenvalues,

$$\langle M \rangle_i = \frac{(\det m)^{1/N} \Lambda^{(3N-F)/N}}{m_i}. \qquad (2.4)$$

For $F < N$, the $\langle M \rangle_i$ exhibit runaway behavior in the massless SQCD limit. For $F = N$, we can approach the massless limit without the vacuum running away. The $\langle M \rangle_i$ can be chosen arbitrarily by choosing the ratios m_i/m_j as $m_i \to 0$, subject only to the constraint

$$\det\langle M \rangle \equiv \prod_i \langle M \rangle_i = \Lambda^{2N}. \qquad (2.5)$$

This is just the quantum deformed moduli constraint that emerges when approaching massless SQCD from massive SQCD, since, as mentioned above, the baryon vevs which usually appear in the constraint are always at zero vev for arbitrary but non-zero quark masses. For $F > N$ we can arrange arbitrary vevs for the mesons as we approach the massless limit, subject only to the rank constraint, namely that only N of the $\langle M \rangle_i$'s can be non-zero.

The results described above are deduced quite concretely. On the other hand duality has to be guessed, and then shown to satisfy a number of consistency conditions. For $F > N + 1$ the infrared behavior of SQCD is believed to have a dual description [1] in terms of an $SU(\tilde{N})$ gauge group, $\tilde{N} = F - N$, with F flavors of quarks and anti-quarks, and with gauge-singlet mesons which are interpolated by the meson operators discussed above. The meson fields of the dual theory are usually denoted by their interpolating operator, the difference in dimension being compensated by a scale μ. The dual effective theory has a superpotential given by,

$$W_{dual} = \text{tr } mM + \frac{1}{\mu}\tilde{Q}_+ M \tilde{Q}_-, \qquad (2.6)$$

at energies far above the $\langle M \rangle_i/\mu$. The strong interaction scale of the dual theory is given by

$$\tilde{\Lambda}^{3\tilde{N}-F} \Lambda^{3N-F} = (-1)^{\tilde{N}} \mu^F. \qquad (2.7)$$

The scale μ appearing in eq. (2.6) and eq. (2.7) is arbitrary in the sense that it does not affect the low-energy regime where duality is expected to hold. Below the dual quark masses $\langle M \rangle_i/\mu$, the dual quarks can be integrated out of the theory and the dual gauge theory undergoes gaugino condensation, resulting in,

$$W_{dual} = \text{tr } mM + \tilde{N}\tilde{\Lambda}^{(3\tilde{N}-F)/\tilde{N}}(\det\frac{M}{\mu})^{1/\tilde{N}}. \qquad (2.8)$$

Note that this is independent of μ when expressed in terms of Λ using eq. (2.7). The case $F = N + 1$ is a somewhat degenerate case of duality with the trivial dual gauge group "$SU(1)$" whose "dual quarks" are just baryons.

The quark masses break all non-abelian chiral symmetries. We will focus our attention on two $U(1)_R$-symmetries, differing only in the charges assigned to the matter multiplets:

$$
\begin{array}{c|cccc}
 & Q_+, Q_- & M & m & \Lambda^{3N-F} \\
\hline
R_v & 0 & 0 & 2 & 2(N-F) \\
R_w & 1 & 2 & 0 & 2N
\end{array}
\tag{2.9}
$$

Both symmetries are anomalous, symmetry transformations result in shifts of the θ angle. These shifts are indicated by assigning spurious charges to the strong scale Λ (see eq. (2.1)). Periodicity of physics in θ then implies that the anomaly breaks the R_v-symmetry to its $\mathbf{Z}_{2(N-F)}$ subgroup, while the R_w-symmetry is broken down to its $\mathbf{Z}_{2N}$ subgroup. The R_v-symmetry is also explicitly broken by m, so that R_v-transformations induce rotations of m. This is indicated by the spurious charge assigned to m. The $\mathbf{Z}_{2N}$ symmetry, though an exact dynamical symmetry, is spontaneously broken down to $\mathbf{Z}_2$ by gaugino condensation.

In this paper we shall consider $F < 3N$ so that SQCD is asymptotically free and has interesting non-perturbative effects.

3 The Type-IIA string theory set-up

From now on we shall work in units in which the string mass scale is set to one, $m_s \equiv 1$. Our SQCD theory can be described as the low-energy limit of D4-branes suspended between Neveu-Schwarz (NS) 5-branes in Type IIA string theory. The ten-dimensional configuration of branes we will consider is depicted in Fig. 1. It is similar to the configurations with D6-branes used to study SQCD in ref. [6] but employs semi-infinite fourbranes as suggested in [23] [28]. All the branes occupy the four dimensions spanned by the x^0, x^1, x^2, x^3 directions and all of them sit at $x^9 = 0$, so these five dimensions have been suppressed in the figure. The x^4 and x^5 coordinates are conveniently paired into a complex coordinate, as are the x^7 and x^8 coordinates,

$$
v = x^4 + ix^5, \quad w = x^7 + ix^8.
\tag{3.1}
$$

These complex dimensions are represented schematically as real dimensions in Fig. 1. (One can consider the imaginary components of v and w to have been suppressed.) The NS 5_v-brane sits at $w = 0$ and fills the v-plane, while the NS 5_w-brane sits at $v = 0$ and fills the w-plane. The two 5-branes are separated in the x^6-direction
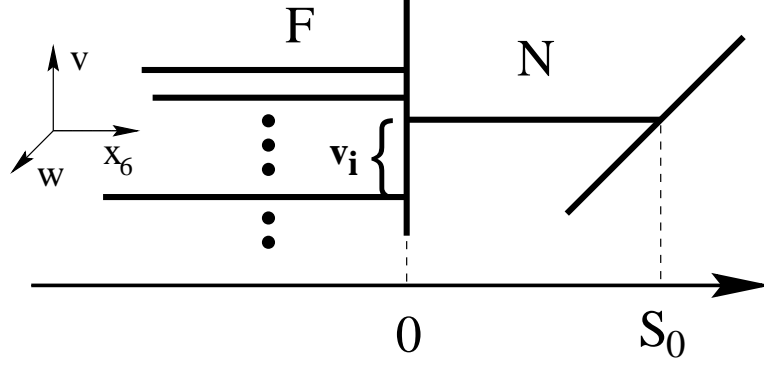
Fig 1. *The classical string theory brane configuration corresponding to SQCD. The positions v_i of the semi-infinite D4 branes determine the bare masses of the quark fields.*

by a distance S_0. N coincident D4-branes are suspended between the 5-branes at $v = w = 0$. F semi-infinite D4-branes at $w = 0$ and $v = v_1, ..., v_F$ come in from $x_6 = -\infty$ and attach to the NS 5-brane.

The low-energy particle content essentially consists of the (nearly) massless strings with ends attached to D4-branes. There are N^2 massless vector fields corresponding to the short strings connecting one of the N finite D4-branes with another. The five-dimensional strip occupied by these states is effectively four-dimensional if S_0 is taken very small, and this is identified with the four-dimensional spacetime of the low-energy limit. The massless vector fields fields form the gauge fields of a $U(N)$ gauge symmetry. The $U(1)$ factor corresponding to the trace of the $U(N)$ group has been argued to be frozen by an infrared singularity on the NS5 branes [23]. However, as has been argued [7], the $U(1)$ can be resurrected by adding a D4-brane at infinity. Even if it is present it is infrared free and decouples from the $SU(N)$ dynamics. We shall ignore the possible $U(1)$ factor in the following. The SQCD gauge coupling is given by

$$\frac{8\pi^2}{g_{SQCD}^2} = \frac{S_0}{g_s}. \tag{3.2}$$

Strings can also connect the F semi-infinite D4-branes and the N finite D4-branes. For small v_i this results in F flavors of quark and anti-quark multiplets, with mass parameters given by

$$m_i = v_i. \tag{3.3}$$

The price of this simple set-up is that all the masses must be taken non-zero. Massless SQCD requires the introduction of more branes. The matter multiplets live in the effectively four-dimensional spacetime as well because they correspond to short strings with one end on the finite D4-branes. There are also short strings connecting the semi-infinite D4-branes with each other. These correspond to unwanted light states living

in *five* (semi-)infinite dimensions! These states do couple to the quark fields living on the four-dimensional boundary of their five-dimensional world, but these couplings are non-renormalizeable (suppressed by powers of m_s) and one can hope that their infrared effects do not interfere with the four-dimensional SQCD dynamics.

4   The M-theory set-up

In the above set-up, the non-perturbative SQCD effects are also non-perturbative in string theory. To get at these effects we go to M-theory, largely following ref. [28]. This is the regime where we take g_s to be large, opening up a new compact x^{10} dimension with radius,

$$R = g_s. \tag{4.1}$$

Now, as long as g_s is perturbatively weak, variations in it do not affect the low-energy limit of the theory, which remains SQCD. Even g_{SQCD} can be kept fixed by varying S_0 with g_s. It is assumed that the very low energy limit is unaffected by taking g_s large enough to enter semi-classical M-theory. We can then use M-theory to solve for the non-perturbative long-distance dynamics of SQCD.

In M-theory, the brane configuration of Fig. 1 becomes "thickened" in the new dimension, so that D4-branes also become 5-branes, but wrapped around the x^{10}-direction. They smoothly connect to the pre-existing 5-branes. The resulting smooth five-dimensional surface has the form $\mathbf{R}^4 \times \Sigma$, where $\mathbf{R}^4$ is a copy of four-dimensional spacetime and Σ is a complex one-dimensional curve in a complex three-dimensional space spanned by v, w and $t \equiv e^{-s/R}$, where,

$$s \equiv x^6 + ix^{10}. \tag{4.2}$$

We can determine the M-theory curve corresponding to Fig. 1 by noting that if the 5_w-brane were removed we would have semi-infinite branes on either side of the 5_v-brane, which is a degenerate case of the $N = 2$ supersymmetric configurations dealt with in [23]. The curve for this case has the form,

$$t \prod_{i=1}^{F}(v - v_i) - \xi v^N = 0, \quad w = 0. \tag{4.3}$$

Now let us include the presence of the 5_w-brane. In M-theory this corresponds to points of the curve Σ with large w and small v, that is w has a simple pole in v. Thus,

$$vw = \zeta. \tag{4.4}$$

Eq. (4.3) and eq. (4.4) define the M-theory curve corresponding to Fig. 1, in the sense that for small R (g_s), the curve degenerates to Fig. 1. This is illustrated in Fig. 2, where the curve is plotted in the plane of x^6 and (the real part of) v for
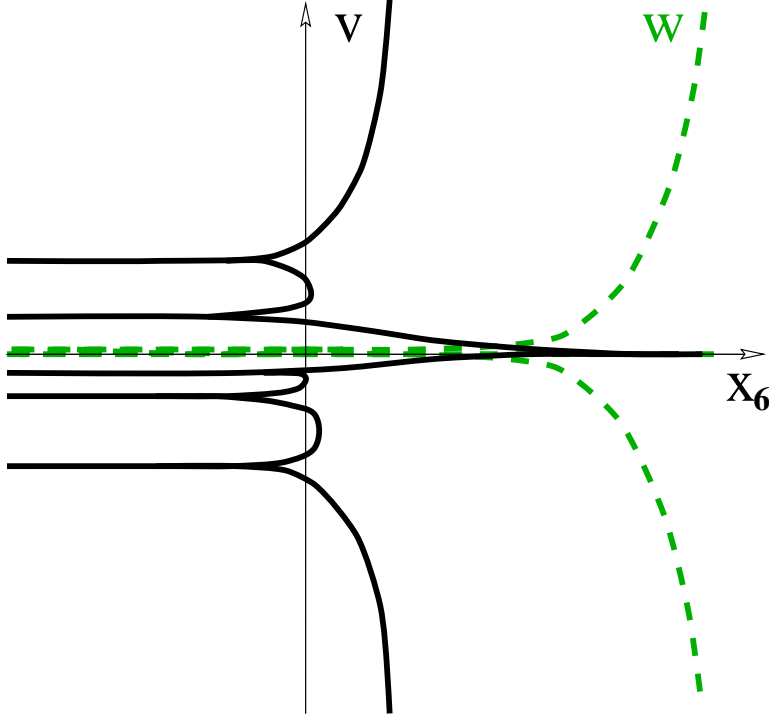
Fig 2. *M theory "thickened" 5-brane configuration which reduces to Fig. 1. in the string theory limit.*

specific choices of ξ, ζ and small R. M-theory has smoothed out the singular brane junctions of string theory in a manner which is non-perturbative in string theory. This smoothing out also contains non-perturbative information about the low-energy limit of SQCD.

The parameters of the M-theory curve can be identified in terms of SQCD parameters as follows [28]:

$$\xi = (\prod_i m_i)^{(F-N)/F}, \quad \zeta = \Lambda^{(3N-F)/N}(\prod_i m_i)^{1/N}, \quad v_i = m_i \ . \tag{4.5}$$

There are some g_s-dependent coefficients on the right-hand sides which we have omitted because they are unimportant for our story. The curve then becomes,

$$t \prod_i (v - m_i) \ - \ (\prod_i m_i)^{(F-N)/F} v^N = 0,$$
$$vw \ = \ (\prod_i m_i)^{\frac{1}{N}} \Lambda^{(3N-F)/N} . \tag{4.6}$$

Rotations of the v and w planes are identified with the R-symmetries discussed in section 2, by assigning R_v-charge two to v and zero to w, and assigning R_w-charge two to w and zero to v.

Now, the symmetries of the asymptotic behavior of the M-theory curve for large v or w correspond to the symmetries of perturbative string theory and SQCD, while the symmetries of the whole M-theory curve correspond to the symmetries of the non-perturbative vacuum. With this in mind one can see the neat fit of the parameter identification above. The asymptotic behavior of the curve for large v is given by

$$v^{F-N}t - (\prod_i m_i)^{(F-N)/F} \sim 0, \quad w \sim 0, \tag{4.7}$$

which is symmetric under R_w and R_v when m is taken to transform spuriously as in eq. (2.9). For large w the curve becomes,

$$w^N t - (-1)^F (\prod_i m_i)^{(F-N)/F} \Lambda^{3F-N} \sim 0, \quad v \sim 0, \tag{4.8}$$

which is not generically $R_{v,w}$-symmetric when m transforms spuriously. Instead the spurious rotations of Λ given by eq. (2.9) result. However we see that an R_v-rotation by angle $\pi/(N-F)$ does leave the curve invariant (when m transforms spuriously), while an R_w-rotation by angle π/N also leaves the curve invariant, corresponding to the non-anomalous $\mathbf{Z}_{2(N-F)}$ and $\mathbf{Z}_{2N}$ symmetries respectively. The exact curve, eq. (4.6), however, does not respect these discrete symmetries, thus reproducing the physics of gaugino condensation.

One can also consider taking a quark mass, m_F, to be very large for a fixed but arbitrary region of v and w. It is straightforward to see that eq. (4.6) then reduces to,

$$\kappa t \prod_i^{F-1} (v - m_i) \quad - \quad (\prod_i^{F-1} m_i)^{(F-1-N)/(F-1)} v^N = 0,$$

$$\kappa \quad = \quad -m_F^{N/F} (\prod_i^{F-1} m_i)^{-N/[F(F-1)]},$$

$$vw \quad = \quad (\prod_i^{F-1} m_i)^{\frac{1}{N}} \Lambda_L^{(3N-F+1)/N}, \tag{4.9}$$

where Λ_L is given by eq. (2.2). κ does not transform spuriously under the R-symmetries and can be absorbed into t by a trivial shift of the s-coordinate origin. We thereby arrive at the curve corresponding to the effective field theory with the massive quark field integrated out. This check, and the check of R-symmetries above, uniquely specify the parameter identification of eq. (4.5).

Having identified the parameters of our M-theory curve, we can express the separation of NS 5-branes in terms of these parameters by studying the $R \to 0$ limit. Comparing the large v behavior of the curve, eq. (4.7), which corresponds to the NS 5_v-brane, with the large w behavior, eq. (4.8), which corresponds to the NS 5_w-brane, we see that the relative separation of these 5-branes in the s-directions, S_0, satisfies

$$e^{-S_0/R} = \Lambda^{3N-F}. \tag{4.10}$$

S_0/R is now generalized from its usage in section 3 to be complex, the imaginary part being an angular separation in the x^{10}-direction. By eq. (2.1),

$$\frac{8\pi^2}{g_{SQCD}^2} + i\theta = \frac{S_0}{R}, \tag{4.11}$$

where the gauge coupling is renormalized at the string scale multiplied by a function of g_s. This reproduces the string theory result eq. (3.2), once we substitute eq. (4.1). At loop-level in SQCD (and string theory) the gauge coupling runs. Witten has pointed out [23] that this feature is reflected in M-theory by the asymptotic logarithmic bending of the curve for large v and large w. This bending can be seen from eq. (4.7) and (4.8). As $R \to 0$ this bending is reduced, and flat 5-branes such as those depicted in Fig. 1 emerge in the limit if S_0 is kept fixed. This is reasonable since it corresponds to taking $g_{SQCD} \to 0$ where the β-function vanishes.

5 Duality from M-theory

We begin by noting that the M-theory curve encodes the non-perturbative vacuum expectation values of the meson operators, eq. (2.4), in a rather suggestive way. (Recall that the baryon and anti-baryon vevs are zero for non-zero quark masses). The SQCD M eigenvalues emerge upon solving for the general $t - w$ relationship in the curve, eq. (4.6) [28].

$$(-1)^F t \prod_i (w - \langle M \rangle_i) \quad - \quad (\prod_i \langle M \rangle_i)^{(F - \tilde{N})/F} w^{\tilde{N}} = 0, \tag{5.1}$$

where $\tilde{N} = F - N$. As explained in section 2, as $m \to 0$ the $\langle M \rangle_i$ display runaway behavior for $F < N$, a quantum deformed moduli space for $F = N$, and the rank constraint on the moduli for $F > N$.

Once the factor of $(-1)^F$ is absorbed into t by a trivial redifinition of the origin of the s-coordinate, eq. (5.1) looks just like part of a curve for a gauge group $SU(\tilde{N})$ with F flavors. Indeed the $v - w$ relationship in eq. (4.6) also obeys this duality as can be seen by rewriting it in terms of $\langle M \rangle_i$ and $\tilde{\Lambda}$ (as given by eq. (2.7) for $\mu = 1$),

$$vw = -(\prod_i \langle M \rangle_i)^{1/\tilde{N}} \tilde{\Lambda}^{(3\tilde{N} - F)/\tilde{N}}. \tag{5.2}$$

Eq. (5.1) and eq. (5.2) are just a rewriting of our old curve, eq. (4.6), but we see that it is also the curve corresponding to the dual gauge group and dual quarks with masses $\langle M \rangle_i$. Of course this is only true if $\tilde{N} > 0$, with $\tilde{N} = 1$ being the degenerate case of duality mentioned in section 2. This M-theoretic duality is the central observation of this paper.

We have found that one and the same curve describes the M-theory configuration associated with the string theory set-up of Fig. 1 as well as the string theory set-up of
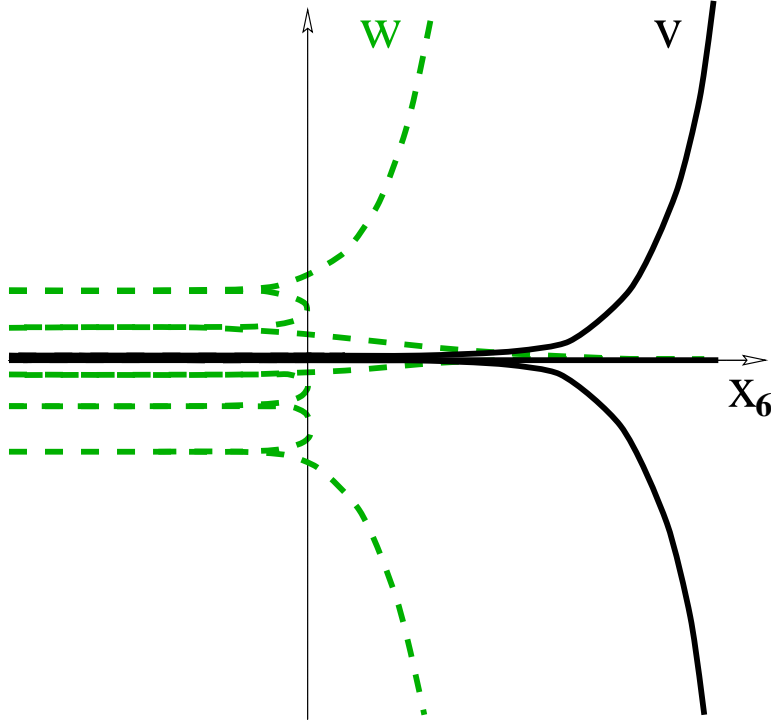
Fig 3. *The classical string theory brane configuration corresponding to the dual of SQCD. The positions w_i of the semi-infinite D4 branes determine the expectation values for the meson field.*

Fig. 3. We have already illustrated in Fig. 2 the sense in which the curve degenerates to Fig. 1 as R becomes small. Clearly it cannot simultaneously degenerate to Fig. 3. To understand this puzzle note that in order to degenerate to Fig. 1 as $R \to 0$ we need to keep m_i and S_0 fixed. By eq. (4.10) this means that $\Lambda \to 0$ exponentially fast. This makes physical sense because it implies that the quarks and gluons are weakly coupled string modes at the string scale, and we expect to see them as short strings in the classical approximation of Fig. 1. However this is not the regime in which dual quarks and gluons are weakly coupled at the string scale, so we should not expect the classical setup of Fig. 3 to be valid. Fig. 3 is the classical string theory set-up corresponding to having dual quarks and gluons being short strings attached to D4-branes, weakly coupled at the string scale. For $3N > F > 3N/2$ the dual theory is also asymptotically free and so the dual quarks and gluons should be weakly coupled at the string scale if and only if $\tilde{\Lambda} \ll 1$, which corresponds to $\Lambda \gg 1$. For $3N/2 > F > N$ the dual theory is infrared free and the dual quarks and gluons are weakly coupled at the string scale if it lies far below the Landau pole, $\tilde{\Lambda}$. Again this corresponds to taking $\Lambda \gg 1$ as can be seen from eq. (2.7). Therefore in both cases we expect to see the dual quarks and gluons weakly coupled at the string scale by taking $\Lambda \gg 1$. Note that this corresponds to $S_0 < 0$ by eq. (4.10), corresponding to reversing the order of the two NS 5-branes. Indeed, if $S_0 < 0$ and $\langle M \rangle_i$ are fixed as $R \to 0$, the curve degenerates to Fig. 3, as illustrated in Fig. 4.

The fact that we must take $\Lambda \gg 1$ to "see" duality has a simple interpretatation in the case $3N/2 > F > N$. Here SQCD becomes strongly coupled in the infrared (if the meson vevs are smaller than Λ). We can think of string/M-theory as providing a supersymmetric ultraviolet cutoff for our field theory. The continuum limit corresponds then to $\Lambda \ll 1$. We can imagine lowering our cutoff by integrating out higher energy physics, until $\Lambda \gg 1$. By this stage we have induced a lot of non-

Fig 4. *M theory "thickened" 5-brane configuration which reduces to Fig. 3. in the string theory limit.*

renormalizeable effects in the effective theory. Let us simply discard these effects. We can hope that the qualitative strong interaction physics is unchanged by this drastic truncation of the theory. This leaves the gauge coupling which is now very large at the cutoff, so we are justified in doing a strong coupling expansion. This is precisely the nature of strong coupling expansions of (non-supersymmetric) QCD formulated on the lattice. Much of the qualitative non-perturbative physics of confinement is thereby reproduced, though quantitative details are not. This is also the situation we find ourselves in. We have $\Lambda \gg 1$, which justifies a strong coupling expansion. The strong coupling expansion of the associated string theory is provided by M-theory. Just as in lattice strong-coupling expansions, M-theory does not provide a *quantitatively* accurate solution of all aspects of SQCD (for example it will not accurately give the mass ratios of hadrons) [29], but it provides us with the correct infrared limit, in particular SQCD duality.

While we have already found that the meson vevs are contained in M-theory, it is disturbing that the low-energy limit of the dual set-up in Fig. 3 does not appear to permit short strings that correspond to the meson fluctuations. These light meson degrees of freedom are required in the dual field theory. Light string modes with the right flavor quantum numbers do appear connecting the semi-infinite branes, but as

mentioned in section 3, these modes live in five dimensions and their coupling to the four-dimensional physics is very weak at low energies. In fact we believe that the problem of the missing mesons is an artifact of our use of semi-infinite branes to set up SQCD and the delicate nature of the decoupling of the unwanted five-dimensional fields. To get around this we must regulate the semi-infinite branes somehow. This problem is discussed for the simple case of equal quark masses in section 6, and indeed the requisite mesons then naturally emerge.

We have shown that by varying parameters of the M-theory curve we can pass smoothly between Fig. 2 and Fig. 4. The analog of this in the string theory limit is passing from Fig. 1 to Fig. 3. Apparently we have moved the 5-branes through each other in this process and changed the number of D4-branes between them. This is just the type of move that has been proposed within string theory, in various contexts [3-12], for deriving field theory dualities. Such moves in string theory involve certain singular intermediate configurations, which in the present case corresponds to the stage at which the two 5-branes intersect. Recalling eq. (4.10) we see that the SQCD coupling formally blows up there. By contrast in our M-theory derivation of the move there is no singular intermediate stage. In this sense M-theory has bought us a rather vivid resolution of the singularities encountered in the string theory moves and a simple derivation of the fact that D4-branes are created between NS 5-branes as they are passed through each other.

6 Duality with finite D4-branes

In order to avoid the pathologies associated with semi-infinite branes and identify the missing meson of dual SQCD we now present a modified brane setup with D4 branes which are of finite extent in the x_6 direction. With a finite x_6 direction we can expect to find a well defined four-dimensional effective field theory on the D4-branes.

There are several choices for cutting off the semi-infinite branes. One possibility which we are not going to explore in this paper is to have them end on D6-branes which are located at large negative x_6 values. Alternatively the D4-branes could end on an NS5 brane at large negative x_6 as depicted schematically in Fig. 5.a. The problem with this setup is that the D4-branes are now free to slide along the two parallel NS 5-branes. This implies a moduli space of vacua parameterized by the positions of the D4-branes with an associated chiral superfield which transforms as an adjoint of $SU(F)$ flavor. The existence of this scalar can also be seen by noting that the theory on the D4-branes between the parallel NS 5-branes is approximately $N = 2$ supersymmetric thus having an adjoint chiral multiplet in addition to the $N = 1$ gauge multiplet. Clearly this theory differs from our target theory SQCD.

However, it is possible to give the adjoint superfield a mass by rotating the NS 5-brane located at large negative x_6 into the w direction as shown in Fig. 5.b. For rotation angles between zero and $\pi/2$ this corresponds to a finite mass for the adjoint
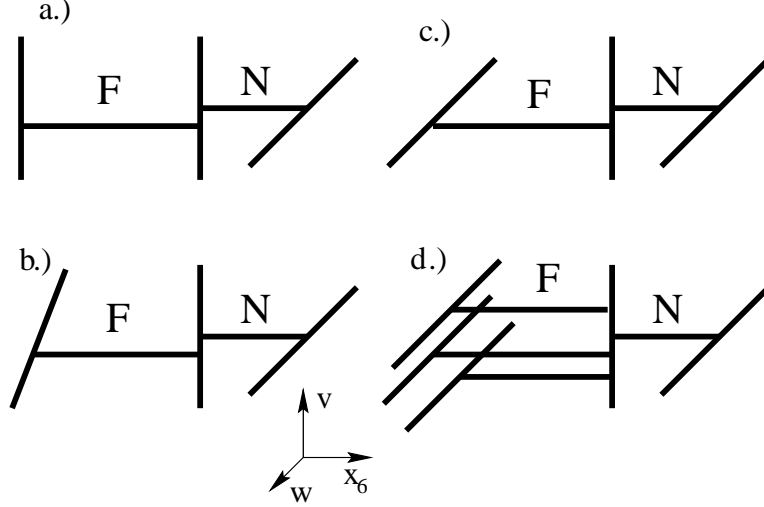
Fig 5. *Different possibilities for terminating the flavor branes at large negative x_6.*

[8] [27]. This mass goes to infinity as the rotation angle is taken to be $\pi/2$ (Fig. 5.c.), leaving us with an effective $N = 1$ gauge theory with no adjoint scalar on the "flavor branes". This is the setup that we want to focus on. At length scales larger than the typical x_6 dimensions the low energy effective theory is SQCD with N colors and F flavors with a weakly gauged flavor group $SU(F)$. By taking the flavor D4-branes longer than the color D4-branes we can make the scale of the $SU(F)$ gauge group exponentially smaller than the scale of the $SU(N)$.

Note that the setup of Fig. 5.c. only allows a common mass for all quarks, but we could easily accomodate more general mass terms by attaching an NS 5-brane to each of the flavor branes individually (Fig. 5.d.). To simplify the considerations, we consider only the common mass case which has all the features we wish to demonstrate.

6.1   The Field Theory

Let us first investigate briefly the properties of the low energy field theory using field theory techniques. The field content and (spurious) symmetries are

$$
\begin{array}{c|cccc}
 & SU(N) & SU(F) & R_v & R_w \\
\hline
Q_+, Q_- & \square\,,\bar{\square} & \square\,,\bar{\square} & 0 & 1 \\
m & 1 & 1 & 2 & 0 \\
\Lambda_N^{3N-F} & 1 & 1 & 2(N-F) & 2N \\
\Lambda_F^{3F-N} & 1 & 1 & 2(F-N) & 2F
\end{array}
\tag{6.1}
$$

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with a tree level superpotential $W = m\mathrm{tr}Q_+Q_-$. The theory has a discrete set of supersymmetric vacua with expectation values for the meson

$$\mathrm{tr}\langle M \rangle = Fm^{(F-N)/N}\Lambda_N^{(3N-F)/N} + Nm^{(N-F)/F}\Lambda_F^{(3F-N)/F}, \qquad (6.2)$$

which can be calculated from gaugino condensation and scale matching relations eq. (2.2). Note that by choosing $\Lambda_F \ll (\Lambda_N, m)$ one can decouple the contribution of the dual gauge group and reproduce the SQCD meson vev eq. (2.4) for the case of a common mass. For $F > N$ the theory has a dual description with an $SU(F - N)$ dual gauge group and F flavors transforming under the weakly gauged $SU(F)$. The dual also has a fundamental meson field which transforms as an adjoint, M_{adj}, plus a singlet, $\mathrm{tr}M$, under $SU(F)$ which are both coupled to the dual quarks in the superpotential

$$W = m \ \mathrm{tr}M + \frac{1}{\mu}[\frac{1}{F}\mathrm{tr}M \ (\widetilde{Q}_+\widetilde{Q}_-) + \widetilde{Q}_+M_{\mathrm{adj}}\widetilde{Q}_-] \ . \qquad (6.3)$$

Classically, this dual theory is an O'Raifeartaigh model which breaks supersymmetry. This is most easily seen by noting that the composite matrix $\widetilde{Q}_+\widetilde{Q}_-$ with the $SU(F - N)$ color indices contracted cannot have an expectation value of rank greater than $F - N$ because the rectangular matrices $\widetilde{Q}_+$ and $\widetilde{Q}_-$ are of rank less or equal to $F - N$. But the equations of motion for M_{adj} and $\mathrm{tr}M$ require $\widetilde{Q}_+\widetilde{Q}_-$ to have rank F. Quantum mechanically, supersymmetry is restored by nonperturbative $SU(F - N)$ dynamics which generates the superpotential

$$W_{\mathrm{dyn}} = \widetilde{N} \det(\frac{M_{\mathrm{adj}} + \mathrm{tr}M/F}{\mu})^{1/\widetilde{N}}\widetilde{\Lambda}_{\widetilde{N}}^{(3\widetilde{N}-F)/\widetilde{N}}, \qquad (6.4)$$

where $\widetilde{\Lambda}_{\widetilde{N}}$ is the scale of the dual gauge group. Minimizing the potential including this term reproduces the supersymmetric vacua of the electric theory. This resurrection of supersymmetry by quantum effects in the dual has a nice interpretation in the string/M-theory picture which will be discussed in the next subsection.

Note that in this discussion we have ignored the "weakly" gauged $SU(F)$ symmetry. The justification for this is not completely obvious, and requires a careful consideration of the scales of the problem. For example, for $F \geq 3/2N$ the $SU(F - N)$ gauge coupling and the Yukawa couplings (with $\Lambda_F \ll m \ll \Lambda_N$) are expected to flow to an approximate interacting fixed point (above m). However when scaling to even lower energies the $SU(F)$ gauge coupling increases and the theory moves away from the SQCD fixed point. Thus our theory is only a good approximation to SQCD and its dual if we look at energies large compared to Λ_F.
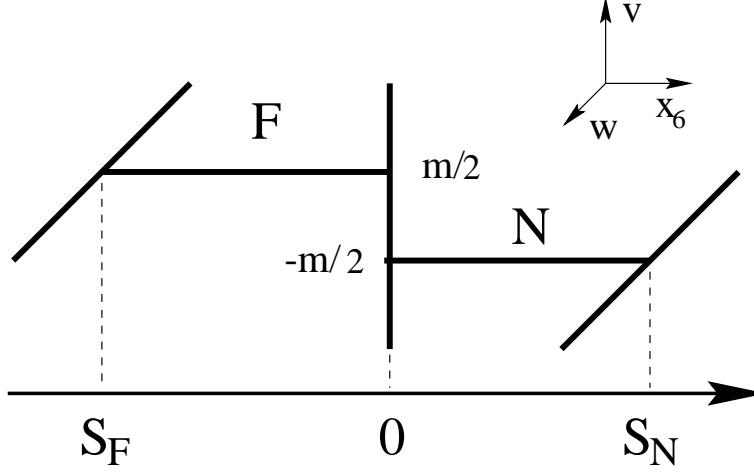
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Fig 6. *The classical string theory brane configuration corresponding to SQCD with a gauged flavor group and a mass m for the quarks. The NS5 brane extending into the v direction is located at $x_6 = 0$ whereas the NS5 branes extending into the w direction are at $x_6 = S_F$ and $x_6 = S_N$.*

6.2 M-theory

In M-theory the brane configuration corresponding to the string theory picture of Fig. 6. is a single 5-brane given by the following curve,

$$t \;=\; m^{F-N}\frac{(v+m/2)^N}{(v-m/2)^F} \tag{6.5}$$

$$w \;=\; m\left[\frac{(m^{F-N}\Lambda_N^{3N-F})^{1/N}}{v+m/2} + \frac{(m^{N-F}\Lambda_F^{3F-N})^{1/F}}{v-m/2}\right] \;. \tag{6.6}$$

Again we ignore a g_s-dependent factor in the w equation.

We can check that the curve has the correct behavior at its infinities

$$
\begin{array}{llll}
i. & v \to \dfrac{m}{2} & t \to \infty, w \to \infty & t \sim \dfrac{w^F}{m^{N-F}\Lambda_F^{3F-N}}, \\[2ex]
ii. & v \to -\dfrac{m}{2} & t \to 0, w \to \infty & t \sim (-1)^F\dfrac{m^{F-N}\Lambda_N^{3N-F}}{w^N}, \\[2ex]
iii. & v \to \infty & t \to 0, w \to 0 & t \sim \left(\dfrac{m}{v}\right)^{F-N} .
\end{array}
\tag{6.7}
$$

One can check that the curve has the spurious R_v and R_w symmetries of the field theory eq. (6.1), that the symmetries are broken to the correct discrete subgroups at the infinities of the curve as expected from instantons in the field theory, and that finally

all symmetries are broken by the whole curve when the spurious transformations are turned off.

As in the case of semi-infinite branes the limit of weakly coupled string theory is reached by taking $R \to 0$ with $\Lambda_F, \Lambda_N \to 0$. If we take

$$\Lambda_N^{3N-F} = e^{-S_N/R} , \qquad \Lambda_F^{3F-N} = e^{S_F/R} \qquad (6.8)$$

where $S_N > 0$ and $S_F < 0$ as well as m are held fixed, we reproduce Fig. 6 with the NS 5 branes positioned at $x_6 = S_F, 0, S_N$ in the limit $R \to 0$.

6.3 Duality

Duality for the string theory setup of Fig. 6. might at first seem a little puzzling. From the field theory we know of the existence of a dual with gauge group $SU(F - N)$ which we might expect to be able to reproduce in string theory by moving the rightmost NS 5-brane to the left of the NS 5-brane at $x_6 = 0$. From arguments based on D4-brane charge conservation on the NS 5-branes one can deduce that the final arrangement has to have $F - N$ D4-branes suspended between the two branes on the right and F D4-branes on the left. The puzzle is that the F D4-branes on the left are suspended between two parallel NS 5-branes which live at different values of v. But this configuration with D4-branes at an angle breaks the remaining supersymmetry.

The resolution of the puzzle is quite simple. We know from the field theory that the dual gauge theory breaks supersymmetry classically, and supersymmetry is only restored through a nonperturbative quantum effect. The string theory brane configuration exactly reproduces the classical result. To see the corresponding quantum effect we should turn to the M theory curve which – because it is defined by the complex curve of eqs. (6.5, 6.6) – preserves supersymmetry.

Recall from the discussion of SQCD with semi-infinite branes that we can find the string theory setup of the dual theory by a smooth deformation of the M-theory curve which corresponds to taking $\Lambda_N \gg 1$. From eq. (6.8) we see that this implies that S_N is negative. Then we take the limit of $R \to 0$ while holding S_N, S_F and the meson vev fixed. This also determines the required scaling of the masses

$$m^{F-N} = e^{S_N/R} , \qquad (6.9)$$

and we see that the masses are driven to zero in the classical limit. Thus the M-theory limit shows that the classical brane configuration which preserves supersymmetry necessarily has zero masses. For any non-zero mass the D4-branes at an angle break supersymmetry. However, in the quantum theory a nonperturbative effect allows the D4-branes to bend while preserving supersymmetry. This conclusion can be supported by calculating the non-perturbative superpotential of the strong field theory gauge dynamics directly from the M-theory curve using an expression for the superpotential recently introduced by Witten [29], given by an integral of the holomorphic 3-form
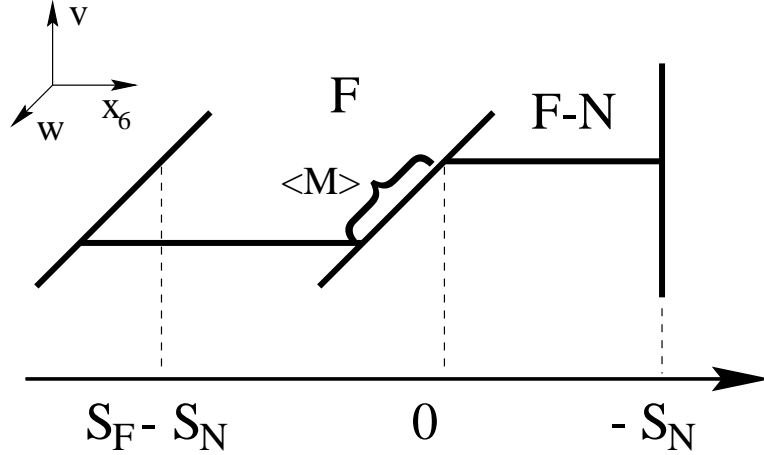
Fig 7. *The classical string theory brane configuration corresponding to the dual of SQCD with a gauged flavor group and an expectation value for the meson field.*

over the curve. We have checked that a straightforward but tedious generalization of Witten's calculation to the case at hand reproduces the field theory result eq. (6.4).

The low energy particle content of the dual theory can now be read off as short strings connecting the various D4-branes. The $SU(F - N)$ gauge bosons arise from strings between the $F - N$ color branes on the right of Fig. 7, and the F flavors of dual quarks are strings connecting color and flavor branes. Since the two NS 5-branes on the left of the diagram are now parallel, the flavor branes suspended between them can slide freely in the w direction. The coordinates of the D4-branes in this direction are interpreted as the meson chiral superfield. Another way of seeing the existence of the meson is to note that the flavor sector is $N = 2$ supersymmetric. Therefore the spectrum of strings connecting the flavor branes not only gives an $N = 1$ gauge multiplet of the $SU(F)$ flavor group but there must also be an adjoint chiral superfield, M_{adj}. This adjoint is coupled to the quark fields in the $N = 2$ symmetric superpotential term $W = \widetilde{Q}_+ M_{\mathrm{adj}} \widetilde{Q}_-$ as required by the field theory duality.

Two issues in the above story need further clarification. Firstly, we have treated the flavor group as a very weakly coupled spectator gauge group and have argued that we can ignore the dynamics of this group. But the gauge coupling of the $SU(F)$ is related to the Yukawa coupling of the meson in the dual by the approximate $N = 2$ supersymmetry of the flavor sector. Thus, if we take the gauge coupling small we are forced to also take the Yukawa coupling small. However this is not a problem since the $N = 1$ duality is only expected to hold at the infrared fixed point. For example, if we take $3N/2 < F < 3N$ so that the infrared regime of SQCD is conformal, then the dual $SU(F - N)$ gauge coupling will also approach a fixed point with associated anomalous dimensions for the quarks (which break the approximate $N = 2$ supersymmetry badly) and the Yukawa coupling will increase to the SQCD

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fixed point value. These anomalous dimensions will also contribute to the running of the asymptotically free $SU(F)$ coupling but in the limit $\Lambda_F \ll m \ll \Lambda_N$ that we are considering the theory can be made to approach the fixed point arbitrarily closely before the $SU(F)$ dynamics becomes important.

Secondly, we seem to be missing the trace of the meson field in the spectrum of our dual. This field is the superpartner of the diagonal $U(1) \subset U(F)$, which is "frozen" by the quantum bending of the NS 5-branes. We believe that the missing singlet might be recovered by considering the above descibed brane setup with an additional D4-brane suspended between the two NS 5-branes which extend to infinity in w. This additional brane does not change the perturbative spectrum and breaks supersymmetry if the non-perturbative dynamics is ignored. In the full quantum theory its presence has the effect of "thawing" the diagonal $U(1)$ and its scalar partner by eliminating the infrared divergence which "froze" it [23, 7].

7   Conclusions

We have derived Seiberg's duality for $N = 1$ supersymmetric QCD from simple M-theory considerations. We expect that this M-theory approach to $N = 1$ duality can be generalized to more complicated theories. It may thereby provide a better understanding of the unifying principle behind duality, as well as the connection between electric and magnetic dual pairs. Our work elucidated such phenomena as the non-perturbative bending of D4-branes and the creation of D4-branes as NS 5-branes are passed through each other. More work needs to be done to satisfactorily resolve the puzzle of the missing flavor singlet meson in the magnetic dual theory arising from our M-theory considerations. It is possible that this may become clearer by studying configurations including 6-branes.

Acknowledgements

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